tion point on the $f(\sigma)$ curve; m, porosity; $q(t)$, resultant "specific" flow rate of fluid through a current tube; $q(t) = Q(t)/ah$ and $q(t) = Q(t)/2\pi h$, respectively, for linear and radial displacement; $Q(t)$, volume flow rate of the phases; a and h, thickness and the width of the bedrock; μ_1 and μ_2, viscosity of the displacing fluid and of the displaced fluid, respectively; $k_1(\sigma)$ and $k_2(\sigma)$, relative penetration factors; $f(\sigma)$, Buckley–Leverett function; D, velocity of propagation of a saturation discontinuity; σ^+ and σ^-, saturation levels, respectively, to the "left" and to the "right" of a discontinuity; t, time; and x, space coordinate.

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NUMERICAL ANALYSIS OF TRANSVERSE STREAMLINING OF A STAGGERED BUNDLE OF TUBES

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The difference scheme of second-order precision [1] is applied to the analysis of transverse streamlining of coaxial circular tubes in a staggered bundle by a viscous incompressible fluid.

We consider transverse streamlining of a bundle of tubes with a circular cross section (cylinders) of the same radius R* (here and henceforth the asterisk will denote a dimensional quantity) staggered parallel in a stream of a viscous heat-conducting incompressible fluid (Fig. 1). The distances between the axes of neighboring cylinders are L* in the longitudinal direction (along the stream) and L_1* in the transverse direction (across the stream). Effects due to the finiteness of the bundle dimensions are eliminated in our calculations by considering a pair of cylinders in one of the inside rows. Such a formulation of the problem will make it possible, with finite dimensions L* and L_1*, to use the conditions of periodicity of the solution at both the entrance to and the exit from the region covered by calculations, and to disregard any possible flow asymmetry even at relatively high values of the Reynolds number.

The problem will be numerically solved by a difference approximation of the Navier–Stokes and energy equations according to the Arakawa scheme [1] of second-order precision for convective terms. The derivatives with respect to time are approximated with central differences. The region ABGHCEF of the mathematical model (Fig. 1) is bounded by the planes of symmetry AB and HC, the planes BG, CD and MN, EF in which the conditions of periodicity are satisfied, and the surfaces AF, GH of cylinders. For calculating the flow around each cylinder, we place its center at the origin of its own polar system of coordinates (r, θ)

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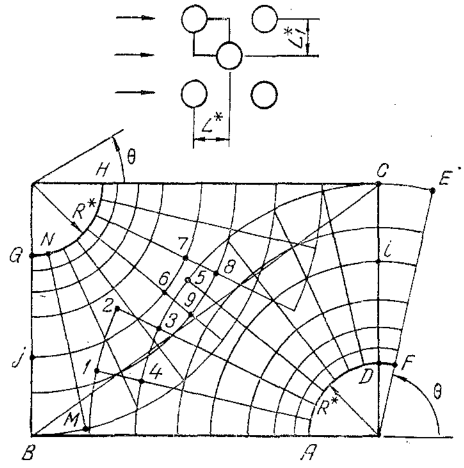

Fig. 1. Region of the mathematical model and difference grid.

and write the corresponding equations in this system. In this way the region of the mathematical model becomes divided into two subregions ABCEF and BGHC, "attached," respectively, to the first (upstream) and the second cylinder. Accordingly, we will call them, respectively, the first and the second flow subregion. It is to be noted that a difficulty of purely computational nature arises in the collocation of the solutions for both subregions at the diagonal BC. Quite evidently, this difficulty is due to the use of polar systems of coordinates and can be eliminated by the use of Cartesian systems of coordinates, as has been proposed in another study [2]. On the basis of experience gathered in the solution of this problem, one can assert, however, that a Cartesian system of coordinates not only requires a large direct-access computer memory but also, with a sufficiently high nodal density of the grid near the surface of a cylinder, is very sensitive to pseudoviscosity due to errors of approximation of differential equations with difference equations (the streamlines near the surface of a cylinder intersect the mathematical cells almost diagonally). One can mention at least two more methods of eliminating these difficulties. In one study [3] a hybrid Cartesian-polar grid was used for solving a problem analogous to this one. In our study only a polar grid was used and the solution was obtained according to a special algorithm of calculating the flow parameters near rectilinear boundaries. In these authors' view, the latter method is more rational: it avoids the difficulties of collocating the solutions at the boundary between regions with different grids and does not require as large a computer memory as required for hybrid grids.

The fundamental dimensionless system of Navier–Stokes and energy equations is written in the form of the system of the vortex transfer equations in vorticity ω, flow function ψ, and static stream temperature T

$$-\omega = \exp(-2\xi)\,\Delta\psi\,; \tag{1}$$

$$-\exp(2\xi)\frac{\partial f}{\partial t} + I + \frac{2}{N_{Re}N_{Pr}{}^{\gamma}}\Delta f = 0\,, \tag{2}$$

where $f \equiv \omega$ and $\gamma = 0$ in the equation of vortex transfer, $f \equiv T$ and $\gamma = 1$ in the equation of energy, $\xi = \ln r$ is the transformed radial coordinate, and $\Delta = \partial^2/\partial\xi^2 + \partial^2/\partial\theta^2$.

The characteristic quantities in Eqs. (1) and (2) are the radius R* of the cylinders and the mean-flow-volume velocity U* of the stream at a section not passing through any cylinder. The dimensionless temperature T is related to the actual temperature T through the equality $T = (T^* - T^*_{w_1})/(T^*_{w_2} - T^*_{w_1})$, where $T^*_{w_1}$ and $T^*_{w_2}$ are constant surface temperatures at, respectively, the first and the second cylinder. Both the Reynolds number and the Prandtl number, as well as the local Nusselt number introduced in the course of calculations, are defined conventionally in terms of the characteristic diameter 2R* of the cylinders.

Let us now formulate the boundary conditions needed for solving the system of Eqs. (1) and (2). In the planes of symmetry, we have

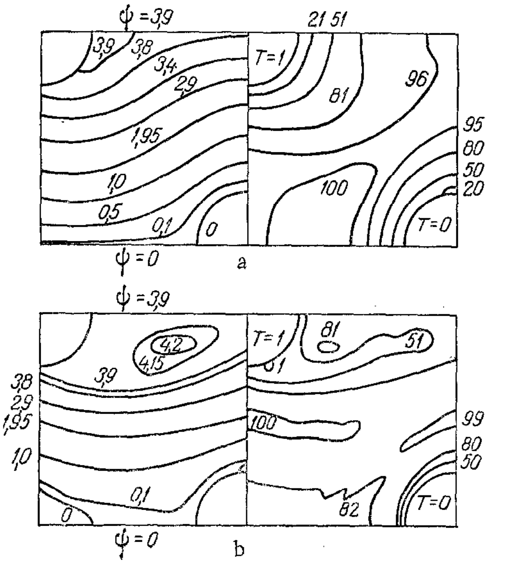

Fig. 2

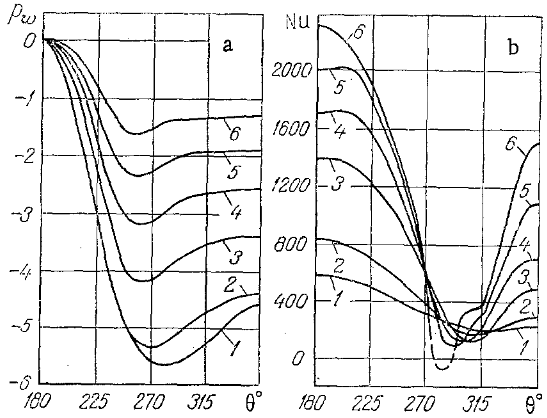

Fig. 3

Fig. 2. Lines of constant flow function ψ and constant temperature T for L = L_1 = 3.9 and N_{Pr} = 0.7, with N_{Re}: (a) 20, (b) 1000.

Fig. 3. Profiles of the pressure (a) and the local Nusselt number (b) over the surface of a cylinder for L = L_1 = 3.9 and N_{Pr} = 0.7, with N_{Re}: 1) Re = 20; 2) 40; 3) 125; 4) 250; 5) 500; 6) 1000.

$$AB: \psi = \omega = \partial T/\partial\theta = 0\ ; \quad HC: \psi = L_1\ ; \quad \omega = \partial T/\partial\theta = 0\ .$$

On the surface of the cylinders we have

$$AF: \psi_{w1} = 0\ ; \quad \omega_{w1} = -2\psi_{w1+1}/h^2\ ; \ T_{w1} = 0\ ;$$
$$GH: \psi_{w2} = L_1\ ; \quad \omega_{w2} = 2(\psi_{w2} - \psi_{w2+1})/h^2\ ; \ T_{w2} = 1\ .$$

Here 1 and 2 refer, respectively, to the first and the second flow subregion; ψ_{w_1+1} and ψ_{w_2+1}, flow functions in grid layers closest to the first and the second cylinder, respectively; and h, taken uniform along both ξ and θ.

The conditions of periodicity are stipulated pairwise along lines EF, NM, GB, DC. Let a point i lie on any of these lines in the first subregion and a point j lie on the corresponding line in the second subregion, both points equidistant from the surface of a cylinder (Fig. 1). Then the conditions of periodicity can be expressed as

$$\psi_i = L_1 - \psi_j\ ; \quad \omega_i = -\omega_j\ ; \quad T_i = T_j - 1\ .$$

We will now establish the boundary conditions in the zone along the diagonal BC for both flow subregions. For convenience, we introduce for L and L_1 the relations L = exp((m − 1)h), L_1 = exp((m_1 − 1)h). We then proceed as follows (Fig. 1).

1. On each radial line in, say, the first subregion we locate the boundary node closest to BC outside the field ABCD. When such nodes on the neighboring radii lie on different circles, then we add the necessary number of nodes in accordance with the requirements of the Arakawa scheme. Having the boundary nodes 1 and 3 on neighboring circles, e.g., we add node 2 for solving Eq. (2) according to the given scheme at node 4.

2. Each thus determined node in the first subregion will lie in some cell of the second subregion BGHC. Let node 5 lie in the cell with nodes 6, 7, 8, 9, for instance. The parameters at node 5 can be calculated from the parameters at the nodes surrounding it through, say, linear interpolation. Then

$$f_5 = l_6 f_6 + l_7 f_7 + l_8 f_8 + l_9 f_9\ , \qquad (3)$$

where f ≡ ψ, ω, or T and where l_i are the interpolation factors, the latter remaining constant throughout the entire calculation process.

Relations analogous to relation (3) can be established for the second flow subregion. The choice of interpolation factors will be the same here as for the first subregion.

The numerical procedure includes solution of the dynamic problem and the thermal problem. Solving the former involves calculation, sequentially in time, of the flow field in both first and second subregions with the boundary conditions appropriately established. The calculation continues until the steady state is reached. As the criterion of steadiness serves an approximately zero derivative of the drag coefficient C_x with respect to time for one cylinder, this coefficient being the sum of its pressure component C_{xp} and friction component C_{xf}

$$C_{xp} = -\left(\int_{\pi}^{\pi/2} p_w \cos\theta d\theta + \int_{3\pi/2}^{2\pi} p_w \cos\theta d\theta\right);$$
$$C_{xf} = -\frac{4}{N_{Re}}\left(\int_{\pi}^{\pi/2} \omega_{w1} \sin\theta d\theta + \int_{3\pi/2}^{2\pi} \omega_{w2} \sin\theta d\theta\right). \quad (4)$$

The pressure at the surface of a cylinder in the first of expressions (4) is found from the equation of motion

$$p_w = -\frac{4}{N_{Re}}\left[\int_{\pi}^{\theta}\left(\frac{\partial\omega}{\partial\xi}\right)_{w1} d\theta - \int_{3\pi/2}^{\theta}\left(\frac{\partial\omega}{\partial\xi}\right)_{w2} d\theta\right] + \text{const},$$

where the constant can be equated to zero without any loss of generality.

After the dynamic problem has been solved, we proceed with the thermal problem and use here the already calculated fields of the flow function. The numerical procedure now involves establishing the boundary conditions and solving the energy Eq. (2) for each subregion of the mathematical model in successive time steps. The calculation continues until the steady state is reached, as a criterion of steadiness serving an approximately zero derivative of the mean-over-the-perimeter Nusselt number

$$R_{Nu_m} = \left(\int_{\pi}^{\pi/2} R_{Nu}\, d\theta - \int_{3\pi/2}^{2\pi} R_{Nu}\, d\theta\right)/\pi$$

with respect to time.

Calculations were made for the variant $L = L_1 = 3.9$ and $N_{Pr} = 0.7$, $N_{Re} = 20$, 40, 125, 250, 500, 1000. The parameters of the computation grid for this numerical experiment were $\Delta t = 0.01$ and $h = \pi/30$. The resulting flow patterns are shown in Fig. 2a, b, where they are described by lines of constant flow function and constant temperature. The calculations have revealed that as the Reynolds number increases, so do the dimensions of the circulation zone in the wake behind a cylinder, and the maximum value of the flow function in the vortex also increases monotonically. The vortex behind a cylinder does not close through the surface of the next cylinder downstream, however, even when $N_{Re} = 1000$.

The profiles of pressure p_w and the local Nusselt number over the surface of a cylinder are shown in Fig. 3a, b. We note that at $N_{Re} = 1000$ there appear fluctuations of the Nusselt number in the vicinity of the least thermally loaded point $\theta = 280°$ (this region is indicated in Fig. 3b by a dash line), although the mean Nusselt number reaches a fully defined steady value even in this case.

The dependence of the drag coefficient C_x, the mean (over the perimeter of a cylinder) Nusselt number N_{Num}, and also the pressure drop Δp across the given segment of the tube bundle on the Reynolds number is shown in Fig. 4, that pressure drop being an important parameter for comparing the calculations with experimental data. It is equal to the sum of the pressure drops across the cylinder surface, along the BA axis, and along the HC axis [3]. For comparison, in Fig. 4 have also been plotted experimental data on Δp [4, 5]. It is to be noted that the Reynolds number was $N_{Re} \geqslant 1000$ in the experiment in study [4] and $N_{Re} \leqslant 21$ in the experiment in study [5]. For this reason, calculations were compared with experimental data only at points corresponding to the extreme values of the Reynolds number (N_{Re} = 20 and 1000).

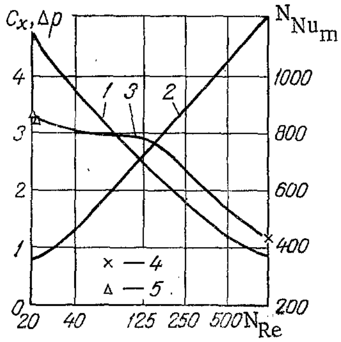

Fig. 4. Dependence of the drag coefficient C_x, the mean Nusselt number N_{Num}, and the pressure drop Δp between two neighboring tubes in tandem in one row on the Reynolds number for $L = L_1 = 3.9$ and $N_{Pr} = 0.7$: 1) C_x, 2) N_{Num}, 3) Δp (calculation), 4) experimental data [4], 5) experimental data [5].

The close agreement between calculated and experimentally determined pressure drop across the intertubular space deserves, therefore, particular attention. This result of purely practical importance raises the expectation that the method developed here can be used for solving problems of streamlining of tubular heat exchange surfaces and optimizing, on this basis, the design of heat exchangers in terms of minimum energy losses. Such problems are now in most cases solved on the basis of experimental studies [6].

NOTATION

R, radius of a cylinder; L^* and L_1, distances between the axes of neighboring cylinders in the longitudinal direction (along the stream) and in the transverse direction (across the stream); r, θ, polar coordinates; ξ, θ, transformed coordinates; U, mean velocity in the largest section (not passing through any cylinder); T, temperature; ω, vorticity; ψ, flow function; p, pressure; t, time; h, step in the difference grid; Δt, time step; N_{Re}, Reynolds number; N_{Pr}, Prandtl number; N_{Nu}, Nusselt number; N_{Num}, mean (over the perimeter of a cylinder) Nusselt number; I, convective terms; Δ, Laplace operator; γ, power exponent which determines the kind of Eq. (2); C_{xp}, coefficient of pressure drag; C_{xf}, coefficient of frictional drag; $C_x = C_{xp} + C_{xf}$, resultant drag coefficient; l, interpolation factor; m, m_1, number of nodes on radial grid lines; an asterisk denotes a dimensional quantity; subscripts: w, conditions at the wall of a cylinder; 1, first cylinder; 2, second cylinder; and i, j, grid nodes.

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